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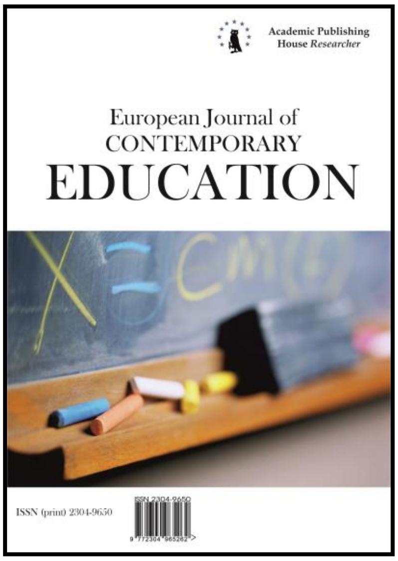

Development of Information Competency in Students during Training in Al-Farabi's Geometric Heritage within the Framework of Supplementary School Education

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Abstract

Information competency is one of the essential qualities of a person living in the information age. It includes skills in information handling in both educational domains and the outside world, as well as readiness and capability to use modern information and communication technologies when involved in various types of information activities. Building and developing information competency in students during teaching and upbringing is one of the primary goals of education. This article describes possible ways of developing it in students within the framework of supplementary education when studying the mathematical heritage of Al-Farabi, one of the greatest scientists of the early Middle Ages, whose fundamental studies made a significant contribution to development of world science. It has been proven that integrated extracurricular classes in geometry and information science are one of the most useful patterns of teaching the scientist's heritage within the framework of supplementary school education. The article highlights the main components of information competency for the most effective organisation of its development process, provides geometric construction problems from the scientist's mathematical heritage, tasks related to his biography and scientific activities, as well as up-to-date educational and information and communication technologies to facilitate the most effective development of information competency in students.

Keywords: Al-Farabi, mathematical heritage, extracurricular work, information competency, geometric constructions, GeoGebra.

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1. Introduction

One of the main educational goals of the modern school is development of key competencies in schoolchildren to ensure their future success in society. The list of key competencies to be formed in school is based on the main goals of general education, the structural representation of social and personal experience, as well as a student's basic activities allowing him/her to gain social experience, and acquire practical skills in modern society. At present, there is no exact list of the competencies to be developed in a general education school. The list proposed in (Khutorskoy, 2003) is most common one. It is characterised by seven items, namely value semantic, general cultural, cognitive educational, information, communicative, social labour and personal self-improvement competencies. These competencies are universal and necessary for a school graduate to achieve success in any domain of public life and professional activities.

Information competency stands out from the group of key competencies of schoolchildren, and is perceived as one of most significant for continuing education, and for solving life and professional problems as a whole.

This competency is an integrative component of the knowledge, skills and abilities of a person to retrieve, analyse, evaluate, organise, present and transmit information, simulated information objects and processes for solving emerging problems in any area of activity, using modern tools of information and communication technologies. It is one of the essential qualities of a person living in the information age.

Its development in schoolchildren is necessary, first of all, for their successful life and professional activities in the future. It is equally important for a successful solution, even at the school level, of personally important educational tasks, such as acquisition of knowledge in subjects being studied, self-education, research and project activities, which is reflected in their academic achievements in general. Therefore, the conditions for developing information competency in students during teaching and upbringing are one of the primary goals of school education.

First of all, it involves strengthening the practical orientation of education through the use of active teaching approaches that build up practical skills, as well as increasing the share of self-motivated work of students for effective organisation of their search and research activities, whose full implementation is possible in a specially organised educational, information and communication space. This is possible within the modern secondary school today, in particular, when organising supplementary education for schoolchildren, where the main purpose is students' acquisition of additional knowledge and skills unforeseen by compulsory educational programmes. With a focus on the free choice and adoption of additional educational programmes by students, it corresponds to their talents and interests. It is also important for their personality development, as well as for a more productive life and activities in society.

The most effective way to develop information competency in students within the framework of supplementary education is a well thought-out set of topics and personal interests of everyone involved in its implementation. The availability of tasks crucial to research that describe the real situation, as well as the practical and cognitive significance of the expected results, is very important. There are many such problems that a student can solve using ICT tools in the mathematical treatises of Al-Farabi, one of the greatest scientists, thinkers and encyclopedists of the early Middle Ages, a native of the Kazakh land, and a world-renowned personality who made a significant contribution to the development of world science. Such tasks will arouse the students' interest, making it possible to verify the practical applicability of acquired knowledge, and thus boost their autonomous cognitive activities.

The predominant part of Al-Farabi's mathematical work has been studied relatively recently, mainly by the well-known Kazakh researcher of the history of mathematics and pedagogy of the Islamic East, A. Kubesov. It is reflected in his works *Mathematical Heritage of Al-Farabi*, *Mathematical Treatises*, *Comments on Almagest by Ptolemy,* which have been highly praised by foreign Farabist scientists (Comments, 1975; Kubesov, 1972; Kubesov, 1974; Garry J. Tee, 1978). However, their application in modern school education has not been studied yet.

The *Book of Mental Skills and Natural Secrets of the Subtleties of Geometric Figures*, whose only manuscript is kept in the library of Uppsala University in Sweden (Kubesov, 1972) stands out from numerous mathematical studies by Al-Farabi. It offers unique algorithms for solving a huge number of geometric construction problems by means of a compass and a ruler, which are important in human practical activities, e.g., land surveying, architecture, engineering, geodesy, etc.

The studies of many of Al-Farabi's predecessors were also dedicated to geometric constructions. A considerable number of works belong to the ancient Greeks, but the oldest book that specifically addresses similar problems is the work of Indian mathematicians of the 7^{th} to 5^{th} centuries B.C.

The centuries-long interest in such problems can be explained not only by their beauty and originality of the solution methods, but above all by their great practical value. Even today, geometric construction problems are of considerable interest, since construction design, architecture, design of various equipment and many other practical tasks are based on geometric constructions. Similar problems also play an enormous role in mathematical development of students. As one of the conceptual lines of a school geometry course, they are a very essential element in teaching geometry, and are an integral part of it.

Al-Farabi's construction problems are distinguished by various applications in practical activities and richness in inter-subject links. They are closely connected with nearly all sections of a school geometry course, which makes it possible to use them as a tool for repetition, generalisation and systematisation of the geometric material being studied. In terms of their formulation and methods of solution, they are the best way to stimulate the accumulation of knowledge in geometry. In addition, they will contribute to the development of students' spatial thinking and boost their exploratory and search skills.

Consideration of a chain of basic constructions proposed by Al-Farabi that lead to the goal, when solved as a kind of algorithm, allows them to be used in upper grades as content-rich material for an information science course. In general, when studying the algorithmisation section, and also when studying application software, it will be more effective if we study the capabilities of modern ICT tools exemplified by such problems. In the process of solving them, a teacher can also effectively build elements of the algorithmic culture in schoolchildren, by systematically demanding a clear sequence of basic constructions.

Since these problems are of an interdisciplinary nature, integrated classes in information science and geometry within the framework of supplementary school education may be one of the most effective teaching approaches (Bidaybekov, 2016).

Al-Farabi's *Book of Mental Skills and Natural Secrets of the Subtleties of Geometric Figures* includes a large number of problems and can satisfy the needs of such extracurricular classes for a sufficient number of special construction tasks. Their study will help deepen students' knowledge of geometry, expand their understanding of construction tasks and possible solutions, and increase their knowledge of algorithmisation. Moreover, their use, along with some historical information about them, will highlight their practical significance, raise students' interest in the material being studied and will contribute to its deep assimilation. The use of modern information and communication technologies for collecting, processing and storage of information will contribute to development of the following skills among students:

- working with various information sources, including the Internet;
- independent searching, extracting, systematising, analysing and selecting information necessary for problem-solving, as well as transforming, saving and transmitting it;
- awareness of information flows, ability to identify their main and necessary points; conscious perception of information published in the global network;
- using a computer and its peripheral devices to work with information in solving these problems, which characterises the student's information competency in active form.

However, despite the rather wide range of pedagogical studies (Yermakov, 2009; Kizik, 2003; Falina, 2007; Trishina, 2005; Moore, 2002; Kamhi-Stein, 1998; Gilinsky, 2008; Kwon, 2011; Cunningham, 2003), issues of developing information competency in students through training in Al-Farabi's geometric construction tasks, together with the scientist's other mathematical achievements, including in the context of supplementary education, within the framework of a general secondary school, have not yet been the subject of a separate study.

The urgency of the problem, its importance and insufficient development have determined the theme of this work.

Research Methodology

A set of methods mutually enriching and complementing each other was used in the research: the method of theoretical analysis conducted with the aim of comprehensive study of the state of the problem in question, revealing the extent to which it has been studied and determining the set of pedagogical conditions for solving it; direct and indirect observation; study of products of schoolchildren's activities.

2. Al-Farabi's Construction Problems in the Context of Supplementary Education of Schoolchildren as a Tool for Developing their Information Competency

2.1 Information Competency and Some Approaches to its Development in Schoolchildren

Information competency is considered as a quality of an individual, including a set of knowledge and skills in performing various types of information activities using ICT tools, together with a value-based attitude towards them.

By analysing studies of different authors (Yermakov, 2009; Kizik, 2003; Falina, 2007; Trishina, 2005), three main components can be distinguished:

- the technological component - knowledge and skills in information handling. It includes main types of information activities in problem-solving using the tools of information and communication technologies, namely, definition, retrieval, integration, management, evaluation, creation and transmission of information. Students must master them and be able to perform them;
- the reflexive evaluative component - understanding and application of knowledge and skills in information handling, both with and without the use of various automatic devices, using a variety of forms and means of communication;
- the motivational value component - the choice of value orientations that reflect a person's motivational intentions, as well as an individual's level of self-awareness.

Highlighting the structural components is advisable both for objective evaluation of the extent of its formation and for the most effective organisation of information competency development.

An effective method of developing it in schoolchildren and their successful mastery of the main types of information activities is the use of various practical information tasks that simulate real-life situations. As noted above, a huge set of such tasks is contained in Al-Farabi's *Book of Mental Skills and Natural Secrets of the Subtleties of Geometric Figures* (Kubesov, 1972). Consisting of 10 books, this work is entirely dedicated to geometric constructions, and as follows from the title "mental skills", it was created to apply geometry to various practical matters and other sciences. The books present unique algorithms for solving a huge number of geometric construction problems by means of a compass and a ruler (this restriction on tools was an indispensable requirement of ancient mathematics). Even for problems that cannot be accurately constructed with the use of these tools, there are algorithms that allow them to be constructed with a practically insignificant error.

Thus, the first book considers elementary constructions using a compass and a ruler. The second book of the treatise is dedicated to regular polygons constructed on an assigned interval, and the third one covers regular polygons inscribed in a circle. The fourth book deals with problems of constructing a circle described around a triangle and regular polygons; the fifth book considers problems of constructing a circle inscribed in a triangle. The sixth book is dedicated to construction of regular polygons inscribed in each other. The seventh book considers equipartition problems of a triangle, and enlarging and reducing it by several times. The eighth book is dedicated to division of quadrilaterals by straight lines satisfying various conditions. The ninth book solves a number of square transformation problems. The tenth book is dedicated to various constructions on a sphere, including division of a sphere into regular spherical polygons equally matched to a construction of inscribed regular polyhedra, whose vertices are the polygon vertices.

All of these problems relate to practical geometry, considers lines and surfaces belonging to specific material bodies. These are lines and surfaces of "a wooden body, if used by a carpenter, or an iron body if used by a blacksmith, or a stone body, if used by a stone mason, or surfaces of the earth and fields if used by a surveyor." All these problems "include purely down-to-earth issues that are the subject of practical arts" (Kubesov, 1974); and they have the nature of activities.

Of course, they are all worthy of study, proof and application in modern school education. It will contribute to both promotion of the scientist's mathematical heritage, to expanding and enriching the system of subject knowledge of students, and to increasing their strength by analysing and repeating the educational material in a new historical context that is interesting and emotionally satisfying for students' perception. The historical context of the training material will greatly strengthen substantiation and persuasiveness of the importance of obtained results.

Based on the foregoing, all of these problems are included in the programme of integrated classes in information science and geometry developed by teachers of the Department of Information Science and Education Informatisation of Abai Kazakh National Pedagogical University, and introduced into a sponsored school within the framework of supplementary education. Studying and working with them will enable schoolchildren to master all the main types of information activities.

Assimilation is most successful with the use of active teaching methods that take into account the psychophysiological characteristics of schoolchildren in designing and carrying out teaching and upbringing aimed at independent mastery of knowledge and skills by students in the process of active mental and practical activities.

The project method was primarily used in teaching Al-Farabi's mathematical heritage within the framework of supplementary school education aimed at development of the students' cognitive and creative skills and critical thinking, and most importantly, the ability to independently construct their knowledge and orient themselves in information space. Most of Al-Farabi's problems were offered to students as project themes for self-study.

It was noted above that all of Al-Farabi's geometric constructions are presented as a clear sequence of actions, which greatly facilitates their computer implementation, thus increasing the efficiency and quality of training. Interactive geometric environments specially designed for use in teaching geometry and allowing the creation of qualitative planimetric and stereometric drawings are of particular interest (Ziatdinov, 2010; Ziatdinov, 2012). GeoGebra software is the most popular of these. It makes it possible to implement all kinds of constructions, including 3D format, and then to dynamically change them, and to build animations. A student can enter equations and manipulate coordinates directly. By offering huge capabilities, GeoGebra allows you to execute geometric constructions using a computer such that when one of the geometric objects of the drawing is changed, the others are also changed, leaving the given relations unchanged. The software can interactively combine geometric, algebraic and numerical representations. Its application in the study of geometric construction problems from Al-Farabi's mathematical heritage will both make construction itself easier, and allow the creation of an interactive dynamic model (Ziatdinov, 2012), whose study provides students with an understanding of the correctness of this construction, and allows them to come up with an idea for proving it and implementing it independently.

2.2 Al-Farabi's Problems on Construction in a Plane

Among planimetric problems presented in the scientist's work, there are a lot of problems on construction of regular polygons of different levels of complexity, including those insoluble with a compass and a ruler.

Regular polygons have always attracted the attention of scientists, builders, architects and designers. Construction algorithms for some of them were considered by Euclid himself, but a major contribution to solving problems of constructing such polygons was made by the German mathematician Carl Friedrich Gauss. He gave all values of n that make it possible to construct a regular n-gon, using a compass and a ruler. These are polygons for which the number of sides is a prime number of the form $2^{2^k}+1$, as well as those obtained from the ones specified by doubling the number of sides. However, it turns out to be impossible to construct regular n-gons that do not satisfy these conditions by means of a compass and a ruler. These are 7-, 9-, 11-, 13-, 14-, 18-, 19-, 21-, 22-, 23 -, 25-, 27-, 28-, ... -gons. Euclid did not consider them. But they are necessary in practice. Therefore, Al-Farabi's studies provide construction algorithms for both heptagons and nonagons using a compass and a ruler. Thanks to these algorithms, the construction is quite simple, although with a certain insignificant error. If necessary, approximation of these algorithms can be shown by justifying them even on the basis of school mathematics knowledge using modern computer tools.

Let us consider some of them.

Here is one of the simplest problems in Al-Farabi's work:

Problem 1. Construct a regular triangle.

He writes "... to build an equilateral triangle on the line *AB*, let us circumscribe circles from each point *A* and *B* as from centers at the distance *AB*. They intersect at the point *C*. Let us join the point *C* with points *A* and *B* by the straight lines *CA* and *CB*. We will obtain an equilateral triangle *ABC*" (Kubesov, 1972).

The text clearly presents a sequence of necessary actions. By consistently performing them, using tools of the environment, one can build a triangle with a given side that will be equilateral (Fig. 1):

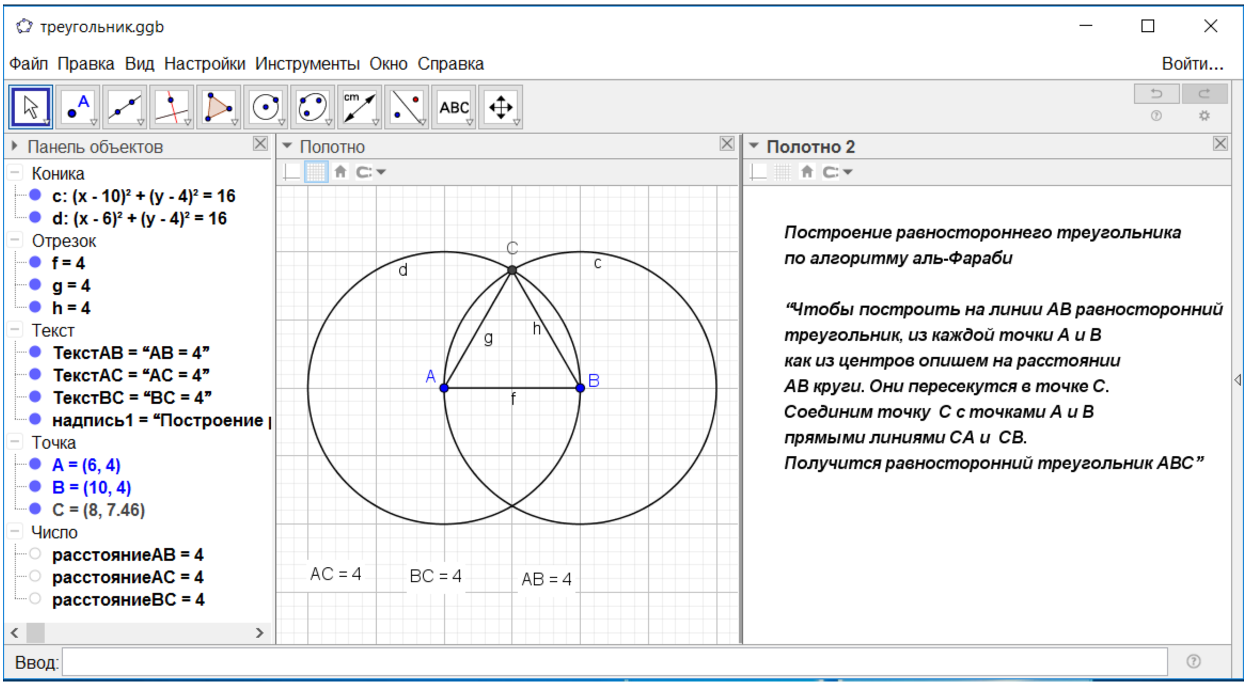

Fig. 1. Construction of an equilateral triangle according to Al-Farabi's algorithm

Thanks to the visual image, justification of the correctness of this construction is quite obvious.

Solution. By construction, *CA* = *AB* and *CB* = *AB* (as radii of circles); consequently, *CA* = *CB* = *AB* and the constructed triangle *ABC* with the sides *CA*, *AB* and *CB* is equilateral.

Students can prove this on their own. A small experiment on the constructed model helps verify it.

Another interesting problem is:

Problem 2. Construct a regular heptagon.

Constructing it with a protractor is not difficult for a student, but it is not easy to do with a ruler and a compass. This problem belongs to the category of construction problems that are insoluble with a compass and a ruler. If ideal accuracy of the drawing is not required by the problem condition, and a small error is not critical, it is possible to construct such a heptagon using a compass and an ordinary ruler on the basis of special algorithms.

Al-Farabi's work provides an algorithm for solving it, which is, of course, approximate with some error, but in the work he does not specify the approximate nature of his constructions, probably because the resulting error was not so significant for practical problems being solved.

He described the solution algorithm thus: "... to build an equilateral heptagon on the line *AB*, let us make the line *BC* equal to the line *AB*, construct the equilateral triangle *DAC* on the line *AC* and circumscribe a circle around the triangle *ADC*. Let us draw a bisecant in it - the line *AE*, equal to the line *AB* - and divide *AE* into two halves at the point *G*, raise the perpendicular *GH* and extend it to the circumference of the circle. Divide *AB* in half at the point *F*, raise in it the perpendicular *FI* equal to the perpendicular *GH*. Draw a circle *ABI* through the points *A*, *B* and *I*, and raise arcs *AK, KL, LI, IM, MN* and *NB*; this is an equilateral and equiangular heptagon" (Kubesov, 1972).

However, like all of Al-Farabi's other algorithms, this algorithm is easy to implement in the GeoGebra software environment described above (Fig. 2).

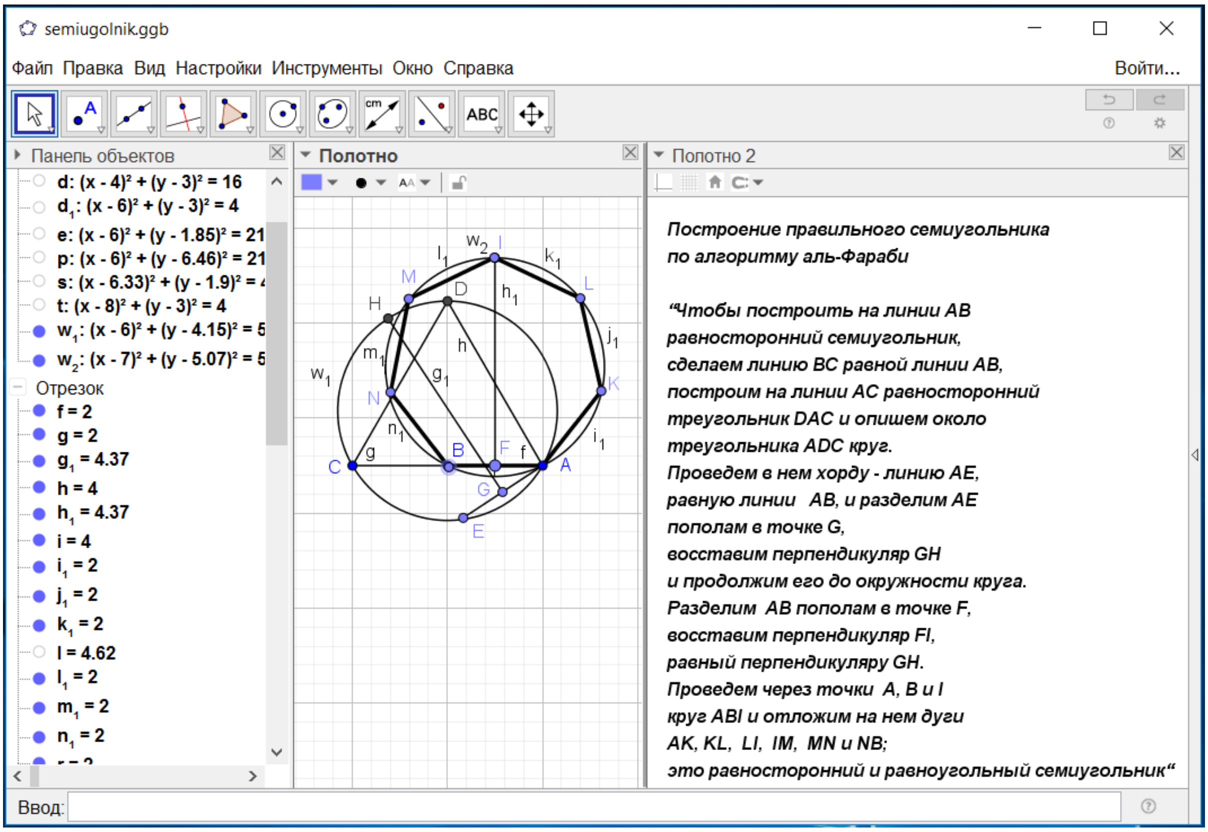

Fig. 2. Construction of a regular heptagon according to Al-Farabi's algorithm

The computer model obtained from constructions based on the described algorithm looks accurate and mathematically exact in this environment.

Students are required both to make constructions according to Al-Farabi's algorithm in this software environment, and to justify this algorithm through a small experiment with the model based on modern knowledge of school geometry.

Solution. According to the constructions based on Al-Farabi's algorithm

$BC=AB; AE=AB; AG=GE;$

$GH \perp AE; AF=FB; FI \perp AB; FI=GH;$

w_1– is a circle circumscribed around a regular triangle with side $AC=2AB$.

w_2– is a circle passing through points *A*, *B* and *I*.

Since according to the construction $AF=FB=AG=GE$, the radii of both these circles w_1 and w_2 are equal to each other: $R_1=R_2$.

R_1, as the radius of the circle circumscribed about the right triangle with side *AC* is equal to $R_1 = 2AB/\sqrt{3}$. From which, $AB = R_1\sqrt{3}/2$.

Bearing in mind that R_1=R_2, the side *AB* of the regular heptagon through the radius of the circle circumscribed about it is determined by the formula: $AB = R_2\sqrt{3}/2$.

Its value to an accuracy of thousandths is equal to $AB \approx 0.866\, R_2$.

On the other hand, according to the formula of the circle circumscribed around the regular heptagon $R_2 = AB/(2\, sin\frac{180}{7})$, we have $AB = 2R_2\, sin\frac{180}{7} \approx 0.868\, R_2$.

Students are asked to find out whether there are other algorithms for constructing a regular heptagon using a compass and a ruler, and what the value of its side is, to compare results for evaluation of Al-Farabi's algorithm.

A series of problems in the scientist's work is dedicated to constructions of some figures inscribed in others using a compass and a ruler. Here is one of them.

Problem 3. Construct an equilateral triangle inscribed in an equilateral quadrilateral.

Al-Farabi proposes the following algorithm: "... let us construct the square *ABCD*, extend the line *DC* to the point *E* and make *CE* equal to *CD*. Construct a semicircle on the line *ED*, make the point *D* the center of the circle and mark the point *G* at the distance *CD*. Next, make the point *E* the center at the distance EG, mark the point *H*, construct *AF* equal to *DH*, connect *B* with *F*, *B* with *H*, *F* with *H*. We obtain the equilateral triangle *BFH* inscribed in the square *ABCD*" (Fig. 3) (Kubesov, 1972).

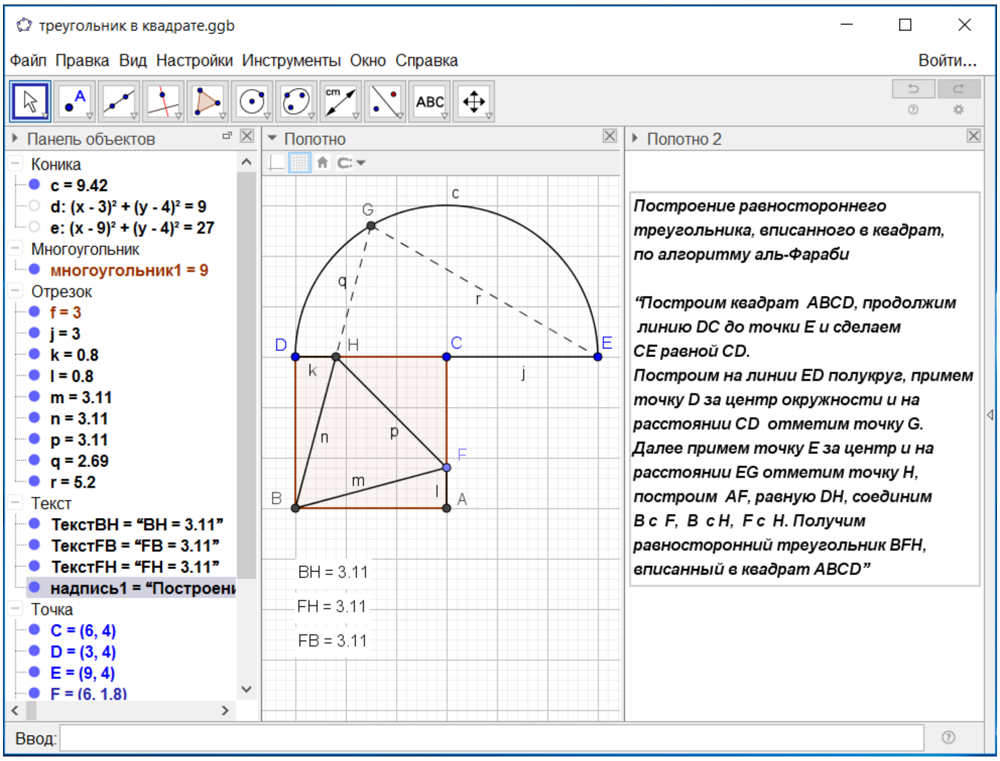

Fig. 3. Construction of an equilateral triangle inscribed in a square according to Al-Farabi's algorithm

The correctness of this construction algorithm is also easy to prove due to the study of a dynamic model built in the GeoGebra environment and based on the knowledge gained earlier from a school geometry course.

Solution. For greater clarity, it is desirable to draw the auxiliary line *DG*.

According to the property of an inscribed angle based on the circle diameter, the angle *DGE* is a right angle; therefore, the triangle *DGE* is a right triangle.

It is easy to find its leg *EG*: $EG = \sqrt{(2CD)^2 - (DG)^2} = \sqrt{3}CD$.

Then the following are defined $DH = 2CD - EH = 2CD - EG = (2 - \sqrt{3})CD$ and

$$BH = \sqrt{(BD)^2 + (DH)^2} = \sqrt{(CD)^2 + \left(2-\sqrt{3}\right)^2 (CD)^2} = 2\sqrt{2-\sqrt{3}}CD.$$

According to the test of equality of triangles by two sides and the angle between them, the triangles *BDH* and *ABF* are equal. Therefore, $BH = BF$.

It remains to determine FH. It is equal to:

$$FH = \sqrt{2(HC)^2} = \sqrt{2(CD-DH)^2} = \sqrt{2\left(CD-\left(2-\sqrt{3}\right)CD\right)^2} = \sqrt{2\left(\left(\sqrt{3}-1\right)CD\right)^2} =$$
$$= 2\sqrt{2-\sqrt{3}}CD.$$

Since all sides of the triangle *FBH* are equal to each other:

$BH = BF = FH = 2\sqrt{2-\sqrt{3}}CD$, this triangle is equilateral.

Work on similar tasks requires students to update the knowledge they have acquired earlier, to master modern information tools and technologies, and to be able to navigate information space in search of the necessary information. They make it possible to synthesise retrieved information, create new knowledge on its basis, raise awareness of the need for additional information and to search for it; select, compare and evaluate it; and organise, process and reproduce retrieved information, which means more than mere captivation of schoolchildren. These information activities contribute to the development of their information competency.

It is noteworthy that these problems, as well as all other construction problems considered in Al-Farabi's work, differ in complexity. This makes it possible to involve nearly all students in the work, ensuring a differentiated approach to their training.

Another project task is team development of interactive models for problems of separating and compiling squares often encountered in practice. Al-Farabi dedicates an entire section to them.

As he writes in his work (Kubesov, 1972), solution of such problems is based on the following principles for constructing a square from two squares:

1. "... if these two squares are equal, let us divide each of them by a diagonal. As a result, we get four equal triangles. Their diagonals are equal to the side of the desired square. If you add these triangles such that they are adjacent to each other with their right angles, you get a square";
2. "... if the squares are unequal, let us construct two rectangles; the length of each of them is equal to the side of the larger square, and the width is equal to the side of the smaller square. Cut each of them in half by a diagonal; we get four equal triangles with sides equal to the sides of the squares; their diagonal is equal to the side of the desired square. If we place a square in the middle with a side equal to the difference of the sides of the two given squares, and arrange the sides of the triangles on its sides, we get one square constructed of squares."

The construction of a square from m^2+n^2 equal squares is based by him on the relation $m^2+n^2= (m-n)^2 +2mn$.

Students are offered the following problems:

Problem 4. "Construct one square from eight equal squares" (you can split the squares, cut them diagonally, move them, or change places).

The solution algorithm proposed by Al-Farabi is quite simple: "... let us construct two squares, each of which consists of four squares. Then we divide them by diagonals; we get four equal triangles. If you combine these triangles such that they adjoin each other with their right angles, you will get a square" (Kubesov, 1972).

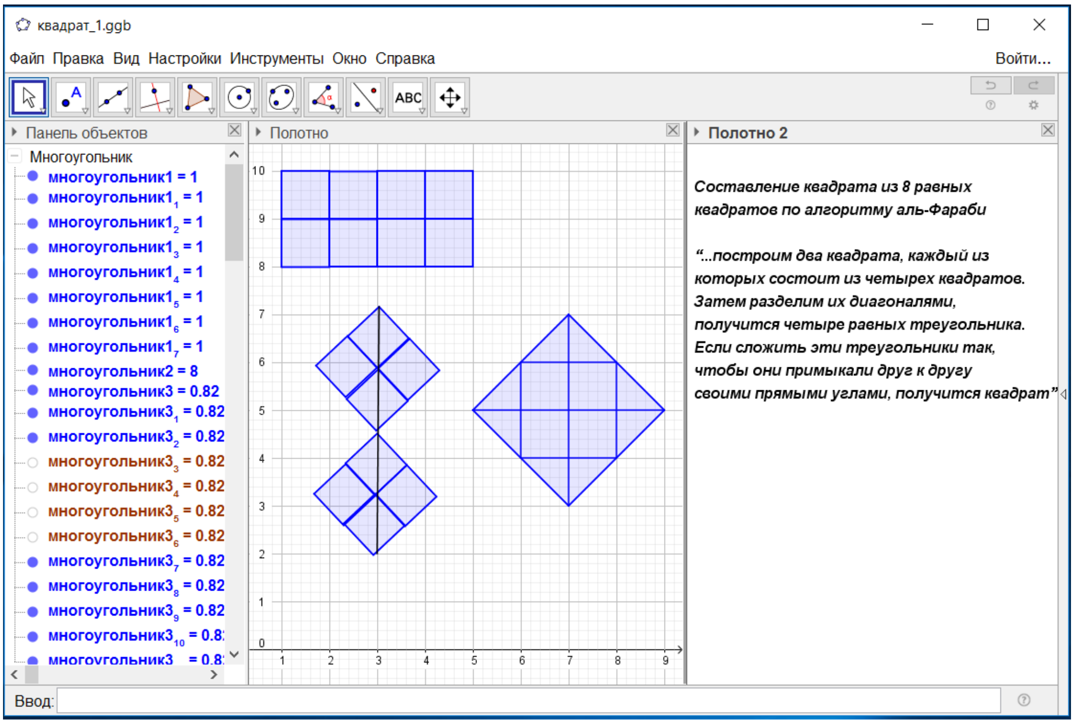

Fig. 4. Compilation of a square from 8 equal squares according to Al-Farabi's algorithm

Problem 5. "There are two squares consisting of nine and four equal squares, respectively. It is required to compile one square from these two squares."

According to Al-Farabi, the solution algorithm is as follows: "... Let us construct two rectangles, each of which consists of six squares. Cut them diagonally; we get four triangles, the long leg of each of which is three, the short leg is two, and the hypotenuse is the root of thirteen. Let us separate a single square from the squares, place it in the middle and adjoin triangles to it with large legs to the side of the square. They will form a square, each side of which is a hypotenuse of the triangles, i.e., the root of thirteen" (Fig. 5) (Kubesov, 1972).

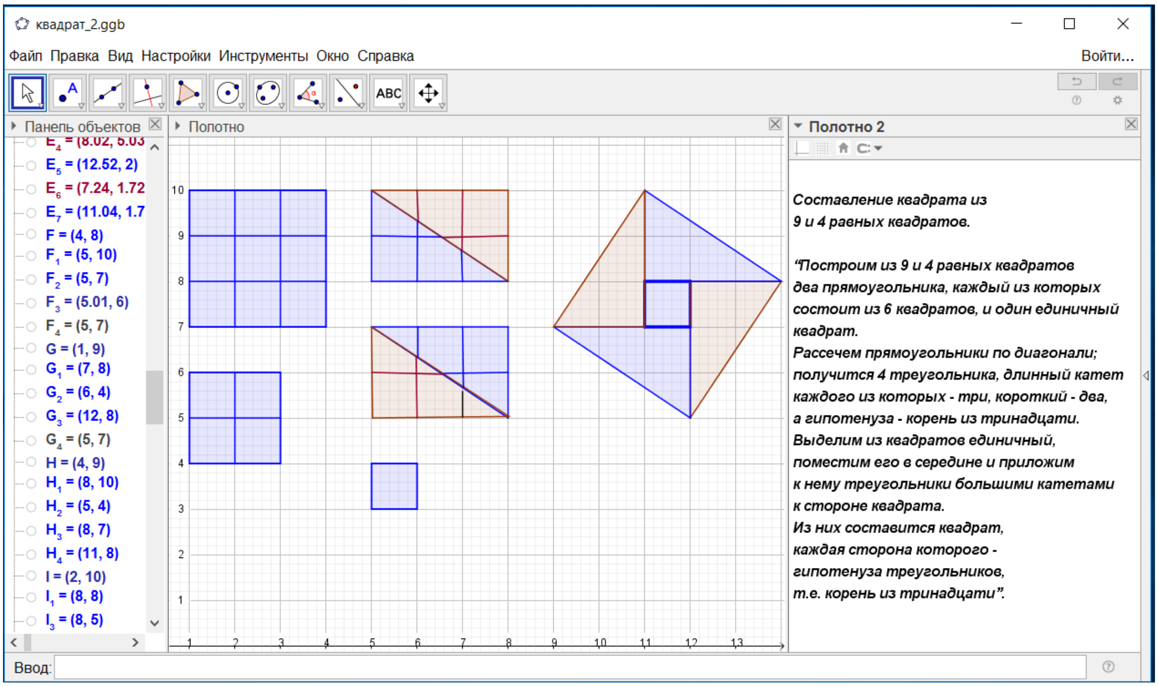

Fig. 5. Compilation of a square from nine and four equal squares according to Al-Farabi's algorithm

However, using the principles of square construction formulated by the scientist, the teams seek the solution on their own, describing it as a sequence of actions that along with the knowledge of GeoGebra capabilities makes it possible to quickly prepare the necessary interactive models without much difficulty (Figs. 4, 5). Only after completion, they are offered a ready algorithm described by Al-Farabi in his treatises for comparison. This contributes, first of all, to building and developing the ability to create information with consideration of a specific problem and to evaluate it, to express the main idea and to give arguments and evidence confirming the correctness of the created information.

2.3 Al-Farabi's Problems on Construction in Space

Students are very interested in problems on construction in space presented in the treatises. These include problems on dividing a sphere into a number of spherical polygons. One of them is as follows:

Problem 6. Divide a sphere into four equal triangles with equal sides and angles, if the sphere diameter is known.

Al-Farabi proposes the following solution: "... if the diameter of the sphere is equal to the line *AB*, let us construct a semicircle on the line *AB*, measure the line *AC* equal to one third of *AB*, draw the line *CD* perpendicular to the line *AB*; it will meet the semicircle *ADB* at the point *D*. Let us take a random point *E* on the circle, make it a pole and draw the circle *FGH* at the distance *BD*, divide it into three equal parts at points *G, H, F* and draw large circle arcs through the pole and through each point *G, H* and *F*, intersecting at the point *J*, and a large circle arc through every two of points of *G, H* and *F*. Then we obtain a sphere divided into four equilateral and equiangular triangles. These are the triangles *JHF*, *JHG*, *FJG* and *GHF*" (Kubesov, 1972).

Using the GeoGebra environment to implement this algorithm greatly simplifies construction, provides an opportunity for more illustrative execution and dynamic manipulation of the drawing parts for better understanding of both the algorithm and obtained results, encourages involvement of students in active cognitive activities by proposing different hypotheses and searching for answers (Figs. 6, 7).

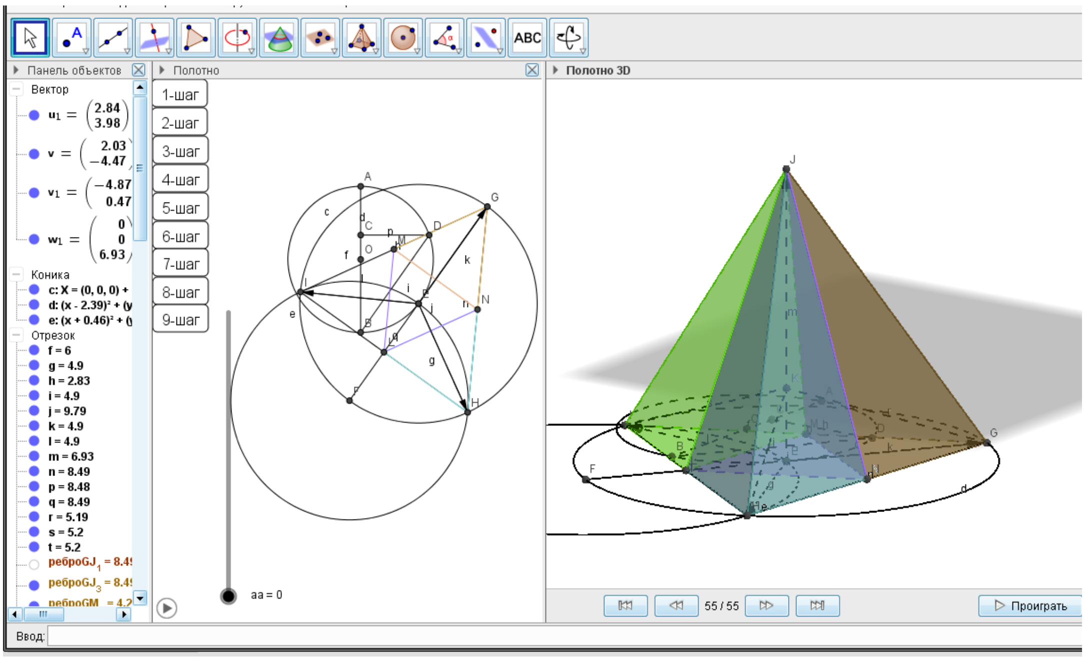

Fig. 6. Division of a sphere into four equal parts according to Al-Farabi's algorithm

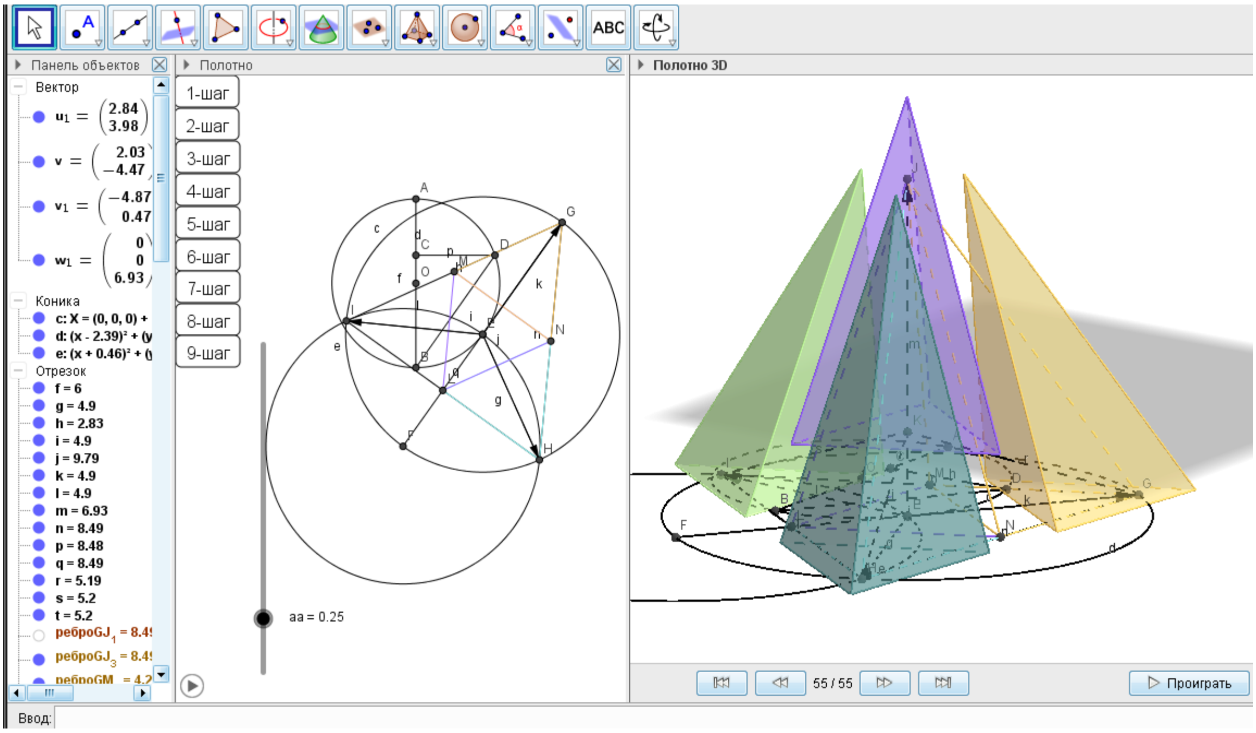

Fig. 7. Animation of parts of a sphere divided according to Al-Farabi's algorithm

Based on theoretical knowledge, such tasks enable students to develop skills in analysis, work with additional sources of information, and boost creativity.

2.4Tasks for Studying the Biography and Scientific Heritage of Al-Farabi

A wider range of schoolchildren become interested in and captivated by problems of Al-Farabi's mathematical heritage through group projects. Participation in the collective struggle for victory and an opportunity to be useful to the team often are critical to awakening their interest.

Group projects within the supplementary education described are diverse. Here is one of them in information science:

Task 1. Develop a web page about Al-Farabi's life and mathematical heritage.

It should contain a biography of the scientist, his mathematical works, a bibliographic index, a photo gallery, the surnames and works of researchers of his scientific heritage, etc. Similar tasks in web programming and design, as well as other types of Internet creativity, are very popular among students.

However, for such a task to contribute to effective development of information competency in students, it must be formulated for a specific individual, taking into account the views and opinions of the students. This is exactly what we have done. It allows them to play the role of an active information converter, without simply repeating other people's texts. Therefore, the following topics were offered to students: *10 Facts from Al-Farabi's Life that Surprised Us*, *The Life and Creativity of Al-Farabi: My Biographical Discoveries in Questions and Answers*, *My Virtual Journey with Al-Farabi*, etc.

Personal importance of the performed task, a high degree of motivation and positive emotions that are a kind of melting pot, in which knowledge and skills acquire the power of competency, are an integrating and connecting element of knowledge and skills in information search acquired by students.

Work organised in this way makes it possible to regulate the activities of students involving the use of Internet resources and contributes to development of their information competency, above all, information search, a component of information competency. In particular:

- the ability to navigate information flows by identifying basic and essential points;
- the ability to systematise, creatively transform, save and transmit retrieved information.

Another assignment within organised extracurricular classes in the mathematical heritage of Al-Farabi involves testing students' knowledge of topics studied earlier (Al-Farabi: the life and mathematical heritage). It is conducted as a game.

Task 2. One student from each team participates in the game. They are offered three topics with three questions each, differing in level of complexity: 20 is an easy question, 30 is a medium-complexity question and 40 is a difficult question. After selecting the topic and specifying the number corresponding to the question number, a video question prepared in advance by the organisers is opened. The participant in the game is required to first press the button and answer this question. If the answer is correct, he/she gets points corresponding to the level of complexity of the question, and has the right to choose the next topic. If he/she makes a mistake, the correct video response is played, and the turn goes to the next participant. When all cells are opened, the game is over, points of each participant are summed up, and victory is awarded to the team whose members earned the most points.

Here are some video questions.

- What is the peculiarity of the algorithm for constructing a regular heptagon using only a compass and a ruler proposed by Al-Farabi? Are there precise algorithms for constructing a regular heptagon using these tools?
- When was the great scientist and thinker Al-Farabi born? Where did he live? Which branches of knowledge were his scientific studies dedicated to?
- How do you scale a scene and animate images in the GeoGebra environment?

Working on this task contributes, above all, to development of the reflexive evaluative and motivational value components of information competency.

2.5 Academic Competition on Al-Farabi's Construction Problems within the Framework of Supplementary Education

An organised school inter-subject academic competition that included geometric construction problems of Al-Farabi helped diversify the activities of schoolchildren when studying Al-Farabi's mathematical heritage within the framework of supplementary education and maintain their interest. The scientist presented these constructions in his work without any proof. Students were required to make the appropriate constructions in the GeoGebra software environment and justify them based on modern knowledge of school geometry. Among them was a problem on dividing quadrangles, which is quite often encountered in practice. For example,

Problem 7. It is required to divide the planar figure $ABCD$ in half by a line passing through one of its angles.

The solution algorithm proposed by Al-Farabi is as follows:

"Take the angle A and draw lines AC and BD intersecting at the point E. Then, if the line BE is equal to the line ED, the line AC divides the figure $ABCD$ in half "(Fig. 8) (Kubesov, 1972).

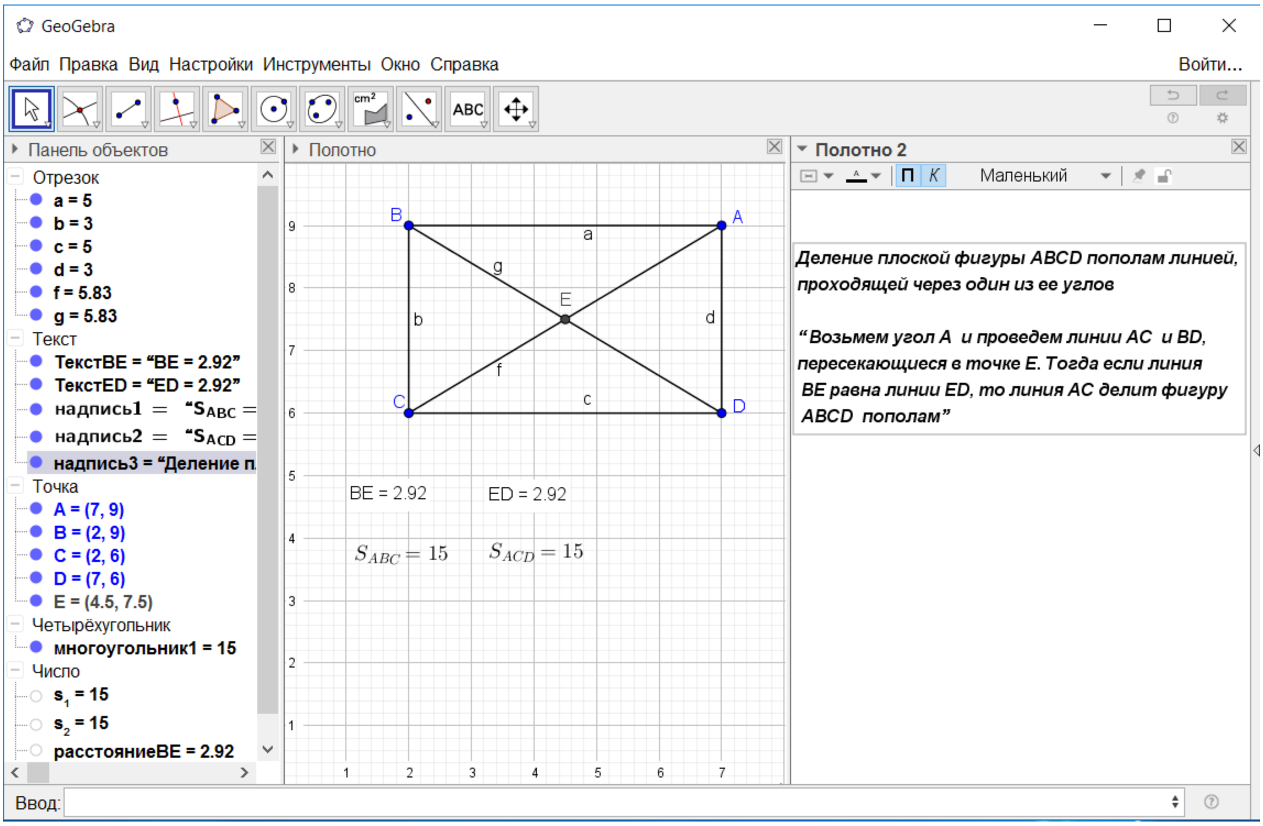

Fig. 8. Division of a planar figure in half with a line passing through one of its angles (when $BE = ED$)

Its proof to be carried out by schoolchildren on their own is simple and is given below.

Solution. Since $BE = ED$, AE is the median of the triangle ABD, and CE is the median of the triangle BCD.

According to the property of the triangle median: "The median breaks the triangle into two equal-sized (with equal areas) triangles." We have

$$S_{ABE} = S_{AED};$$

$$S_{BCE} = S_{ECD}.$$

Then $S_{ABC} = S_{ABE} + S_{BCE} = S_{AED} + S_{ECD} = S_{ACD}$, i.e. $S_{ABC} = S_{ACD}$.

The second case is also possible, when $BE \neq ED$.

The algorithm for solving this problem is described by Al-Farabi as follows:

"If BE is not equal to ED, let us divide BD in half at the point G, draw the line GH through it parallel to the line AC, connect A with H. Then the figure $ABCD$ will be divided in half by the line AH" (Fig. 9) (Kubesov, 1972).

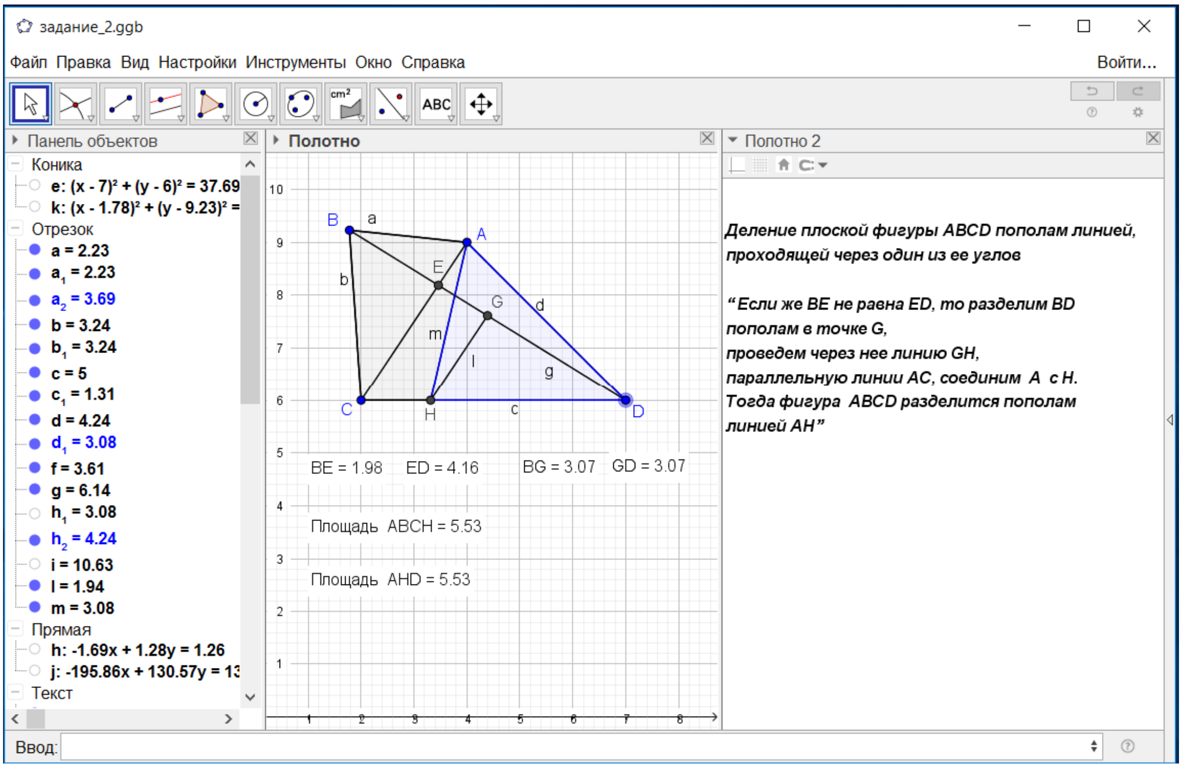

Fig. 9. Division of a planar figure in half by a line passing through one of its angles (when $BE \neq ED$)

Solution. The following property of the triangle median is used for the proof: "The median breaks the triangle into two triangles with equal areas." (These triangles have a common height and equal bases).

The median AG divides the triangle ABD into two triangles of the same area $S_{ABG} = S_{AGD}$. Similarly, the median CG divides the triangle BCD into two triangles of the same area $S_{BCG} = S_{GCD}$.

Then, $S_{ABCG} = S_{AGCD}$.

Let us write both parts of this expression:

$S_{ABCG} = S_{ABCH} - S_{FCH} + S_{AFG}$;

$S_{AGCD} = S_{AHD} - S_{AFG} + S_{FCH}$.

Based on the properties of triangles: "All triangles with a common base, and with vertices positioned on a line parallel to the base are of equal size." We have $S_{ACH} = S_{ACG}$, as triangles ACH and ACG have the common base AC and vertices positioned on the line GH: $GH \parallel AC$.

Triangles ACH and ACG have the common part ACF, consequently, $S_{FCH} = S_{AFG}$.

Therefore,

$S_{ABCG} = S_{ABCH} - S_{FCH} + S_{AFG} = S_{ABCH}$;

$S_{AGCD} = S_{AHD} - S_{AFG} + S_{FCH} = S_{AHD}$,

Consequently, since $S_{ABCG} = S_{AGCD}$, then $S_{ABCH} = S_{AHD}$.

There is an interesting problem among those on division of quadrilaterals considered by Al-Farabi. The problem concerns dividing a square in half, leaving a path of a given width in it. It is used in surveying, for example:

Problem 8. Divide the square $ABCD$ in half, leaving a path of width DH.

For this purpose, the scientist proposes the following algorithm: "continue CA in its direction to M such that MA is equal to CH; continue BA in its direction to L, from the center C at the distance CM draw a circle intersecting the line BA at the point L, and connect L with C. Measure LK equal to CH, draw the line KEG parallel to the line AL, and draw HF parallel to the line DB. Then, the figure HE is not equal to the figure EB" (Fig. 10) (Kubesov, 1972).

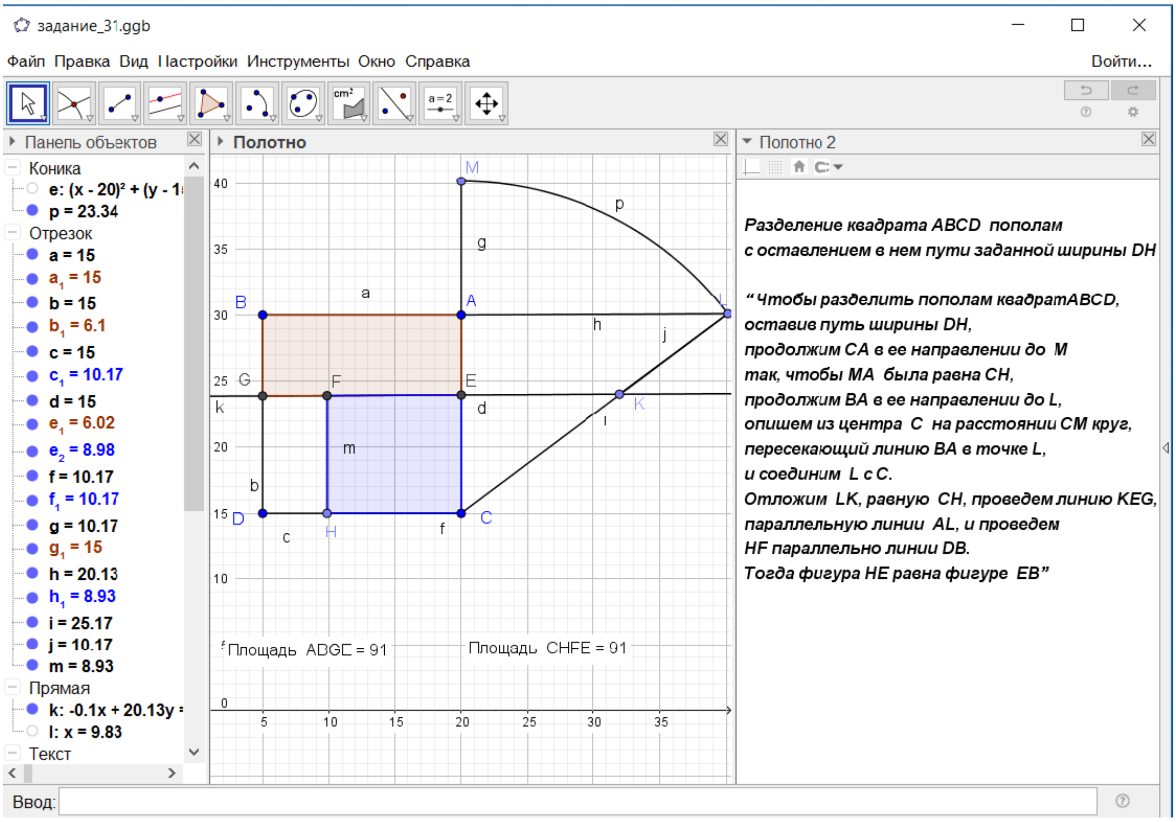

Fig. 10. Division of the square $ABCD$ in half, leaving a path of the given width DH

Solution. The proof is based on a property that follows from the similarity of triangles. Triangles ALC and EKC are similar. The similarity results in $\frac{AC}{EC} = \frac{LC}{KC}$;

$$\frac{AE + EC}{EC} = \frac{CH + KC}{KC};$$

$$1 + \frac{AE}{EC} = 1 + \frac{CH}{KC};$$

$$\frac{AE}{EC} = \frac{CH}{KC};$$

$$CH \cdot EC = AE \cdot KC = AE \cdot BD;$$

$$S_{HE} = S_{EB}.$$

Justifying the correctness of the algorithms considered above requires knowledge of a set of geometric data.

This knowledge and awareness of the capabilities of this software environment needed for implementing the geometric construction problems of Al-Farabi being considered contribute both to consolidation of previously acquired knowledge of geometry and information science, its generalisation and systematisation, and build up and develop the technological and reflexive evaluation components of information competency. Setting a task containing a cognitive issue that is personally meaningful to a student, with a clearly defined incentive (for example, you are a surveyor, and you need to correctly divide a land plot) motivates the student to perform the task, and contributes to development of the motivational value component of information competency.

Thus, skillful use of active methods of teaching Al-Farabi's problems during teaching and upbringing activities within the framework of supplementary education leads to a qualitatively new level of students' skills, ensuring effective development of their information competency.

Google Drive was used to organise students' collaboration on tasks. All participants were previously registered in the service, and assignments were also uploaded there. At present, cloud services allow a user to efficiently solve his/ her problems by using free online office software for processing and storing text and table information, and preparing presentations. The opportunity to work with files directly in the browser interface without downloading them, and having access to them at any time and from any gadget, including mobile devices that are now widely used by young people, have made it possible to effectively organise students' collaboration.

3. Conclusion

To sum up, we would like to stress the vital importance of supplementary education in the general education system in the context under consideration, as an environment for effective teaching of Al-Farabi's geometric heritage. Modern educational technologies used to motivate, stimulate and activate search and cognitive activities of students, which contributes to development of their information competency, as well as communication skills and the ability to reasonably defend their opinions. They boost students' interest in information science and geometry through joint activities, and increase interest in national history and the heritage of the great scientist.

Practice shows the impact of teaching students about Al-Farabi's mathematical heritage in the context of supplementary education on the quality of their academic performance in general. They are more successful in training, and participate in various research competitions and projects that require skills in handling various sources and types of information. They are able to independently retrieve, extract, systematise, analyse and select the necessary information to solve the problems they face. They can creatively transform, save and transmit it using modern ICT tools, draw conclusions on its basis, present an information product and apply it in practice.

Issues related to teaching problems of Al-Farabi's mathematical heritage require further theoretical and experimental research. In particular, it is necessary to determine the criteria, indicators and levels of development of students' information competency in order to objectively evaluate it, as well as to explore opportunities for introducing other mathematical achievements of the scientist in the educational system, and to identify conditions for their effective use in the educational process, both to promote them and develop information competency in students.

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